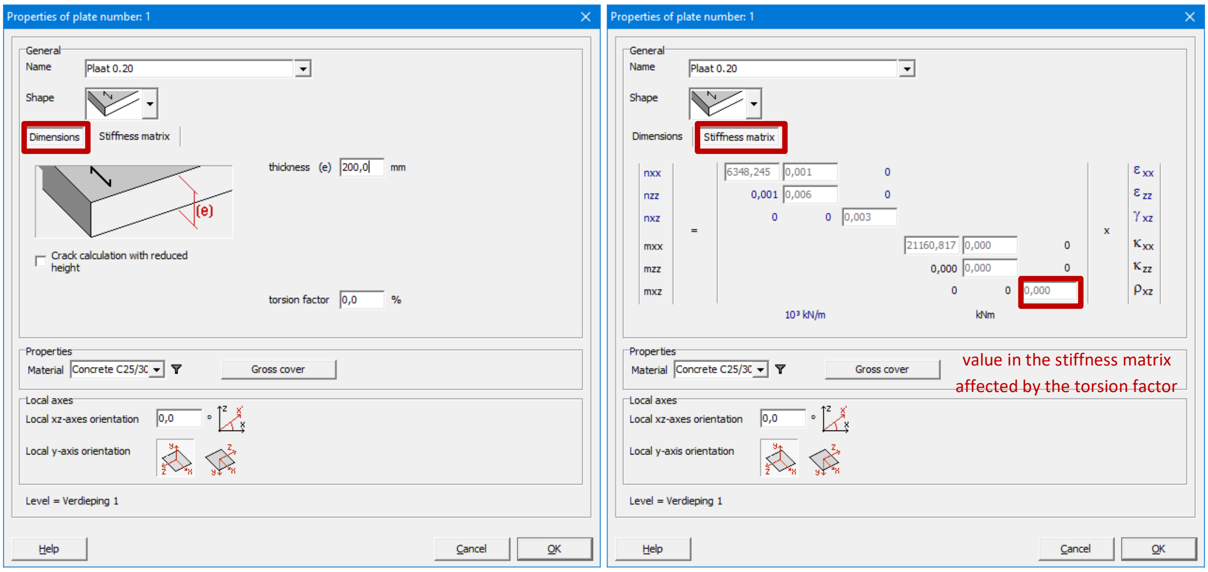
Torsional rigidity of one way slabs

The purpose of this document is

- to explain what the torsion factor τ for one way slabs in Diamonds represents
- to indicate which value you should take for the torsion factor.
- and to illustrate how the torsion factor first into the stiffness matrix.



The document 'The stiffness of plates'[1] explains how a stiffness matrix $[K]$ looks like and how it is determined.

$$[K] = \begin{bmatrix} d_{xx} & d_{\upsilon} & 0 & 0 & 0 & 0 \\ d_{\upsilon} & d_{zz} & 0 & 0 & 0 & 0 \\ 0 & 0 & d_{xz} & 0 & 0 & 0 \\ & & & D_{xx} & D_{\upsilon} & 0 \\ & sym & & D_{\upsilon} & D_{zz} & 0 \\ & & & 0 & 0 & D_{xz} \end{bmatrix}$$

With the goal of this document in mind, we are particularly interested in **the torsional rigidity D_{xz}**.

- The torsional rigidity D_{xz2} of a two directions slab is (Source: Plates and FEM, equ. 21.1):

$$D_{xz2} = \frac{Ee^3}{12(1-\upsilon^2)} \cdot 0.5\,(1-\upsilon)$$

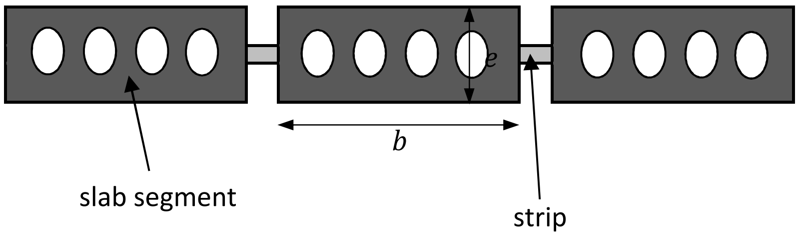
- Since a one direction slab only has a different stiffness in both directions, the torsional rigidity D_{xz1} of a one direction slab is calculated as the mean value of both directions (Source: Plates and FEM, equ. 21.12):

$$D_{xz1} = 0{,}5 \cdot G \cdot i_{av} = 0{,}5 \cdot G \cdot \frac{(i_{xz}+i_{zx})}{2} = 0{,}5 \cdot \frac{E}{2(1+\upsilon)} \cdot \frac{(i_{xz}+i_{zx})}{2}$$

[1] http://buildsoftsupport.com/knowledge-base/stiffness-matrix/

With G the shear modulus, i_{xz} and i_{zx} the torsion constants in respectively the x'-direction and z'- direction smeared out over with width of one floor segment.

An one direction slab in Diamonds has no bearing capacity in the z'-direction, so $i_{zx} \approx 0$. The torsional constant in the x'-direction i_{xz} is derived by considering the slab as a sequence of slab-segments with thickness e and width b. All slab segments are connected by a strip with thickness $e/1000$.



One slab segment is one hollow-core slab.

The torsional constant for each slab segment (neglecting the holes) equals (Source: Berekening van constructies, deel I, §2.4):

$$i_{xz} = \frac{e^3 \cdot b}{16}\left[\frac{16}{3} - \frac{3{,}36e}{b}\left(1 - \frac{e^4}{12b^4}\right)\right]$$

Thus the torsional rigidity D_{xz1} for a one way slab becomes:

$$D_{xz1} = 0{,}5 \cdot G \cdot \frac{i_{xz}}{2} = 0{,}5 \cdot \frac{E}{2(1+\upsilon)} \cdot \frac{i_{xz}}{2}$$

The **torsion factor τ** is the ratio of the torsional stiffness D_{xz1} of a one way slab to the torsional stiffness D_{xz2} of a two way slab.

$$\tau = \frac{D_{xz1}}{D_{xz2}}$$

Or:

$$D_{xz1} = \tau \cdot D_{xz2}$$

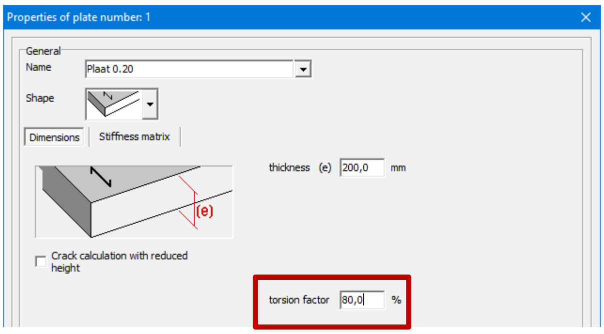
So in Diamonds the torsional stiffness D_{xz1} of a one way slab equals the torsion factor multiplied by the torsional stiffness D_{xz2} of a two way slab with the same thickness.

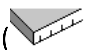
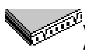
In order to suggest a meaningful value for the torsion factor τ, we calculated the torsional rigidity of a one way slab D_{xz1} for different width b to thickness e ratio's. The torsional rigidity of a one way slab D_{xz1} is divided by the torsional rigidity of a two way slab with the same thickness in order to obtain the torsion factor τ.

b/e [-]	D_{xz1} $[kNm]$	τ [%]
1	3694	42%
2	6004	69%
3	6909	79%
4	7367	84%
6	7825	90%
10	8193	94%

Conclusion:

- If the slab segments have a width to thickness-ratio of 1, already 42% of the torsional rigidity of a two way slab is reached.
- If the slab segments have a width of 60cm and a thickness of 20cm ($\frac{b}{t} = \frac{60}{20} = 3$), 79% of the torsional rigidity of a two way slab is reached.
  If the slab segments have a width of 60cm and a thickness of 15cm ($\frac{b}{t} = \frac{60}{15} = 4$), 84% of the torsional rigidity of a two way slab is reached.
  So a meaningful value for the torsion factor for one way slabs is $\pm$ 80%.



Note: for preslabs and hollow core slabs ( and ) the torsional rigidity is calculated assuming a width/ thickness ration $\frac{b}{e} = 3$.

References:

- J. Blaauwendraad, *Plates and FEM, Surprises and Pitfalls*, Springer, 2010, §21.2.2
- D. Vandepitte, *Berekening van constructies, Deel I*, E. Story-Scientia Gent, 1979, §2.4