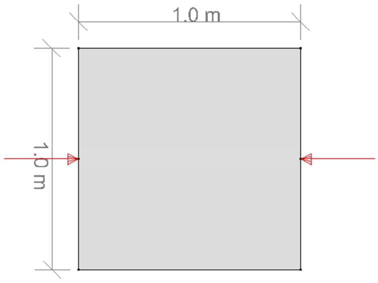
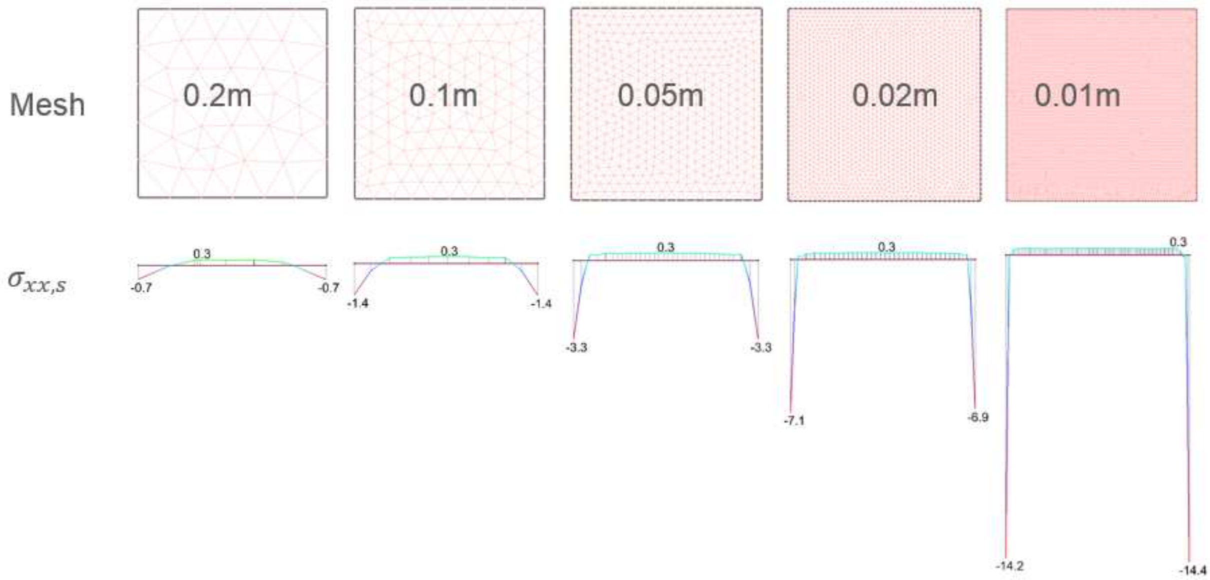
Handling peaks

1. Why do peaks arise?

Consider a square plate (side 1m) compressed by two opposed loads of 100kN[1]:



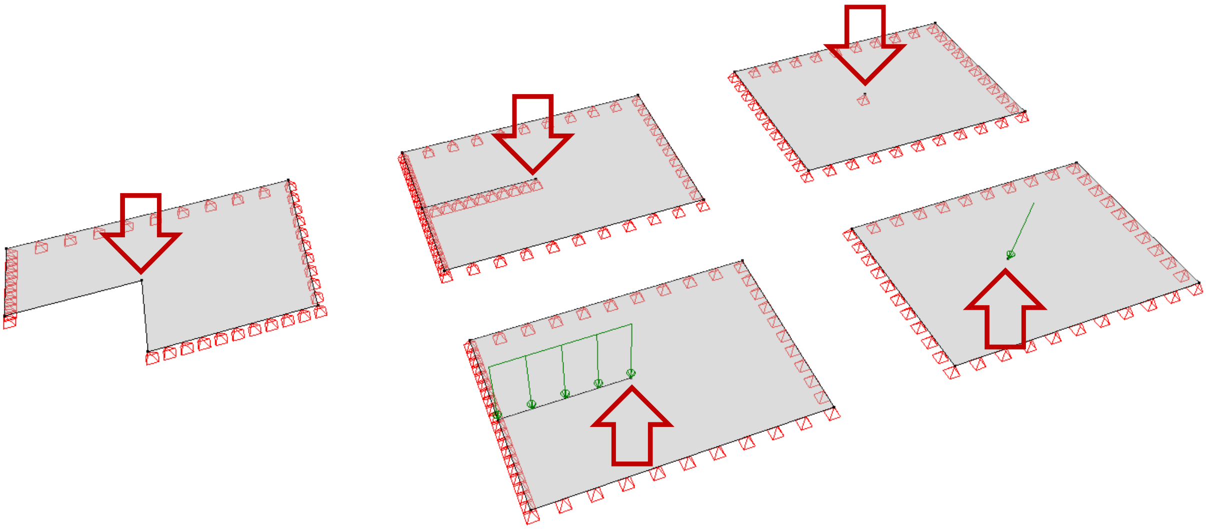
Using Diamonds (= FEM software) we calculate the stress $\sigma_{xx,s}$ for different mesh fineness's:



Conclusions:

- It is expected in a FE analysis that mesh refinement would make the stress result converge to its final correct value. That is the case in the middle of the plate, where you find the constant value of 0,3N/mm². But it is not the case at the borders, where **singularities** occurs.
- **Singularities are inherent to FE analysis! There is no way to avoid them**.
- Typical examples where singularities will occur:
    - Inner corners
    - At the end of a line support or line load
    - Above a point support or point load

[1] This test is called 'the Brazilian splittings test'.



2. How to handle peaks?

Eurocode doesn’t give practical rules to handle peaks. The only ‘rule’ it mentions, is one to spread the bending moment above a middle support of a continuous slab (see EN 1992-1-1 §5.3.2.2 (4)).

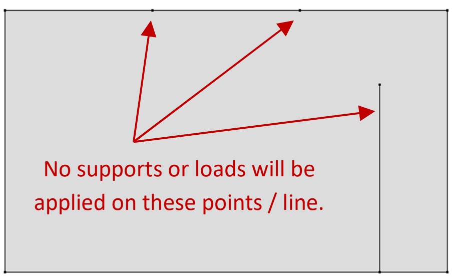
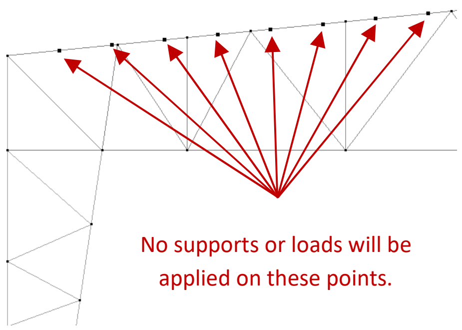
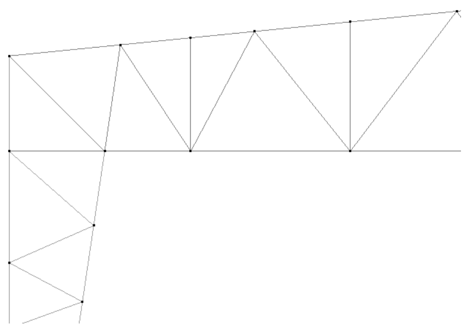
The paragraphs below given an overview of different measures to reduce the value of the peaks.

2.1. Method 1: modelling

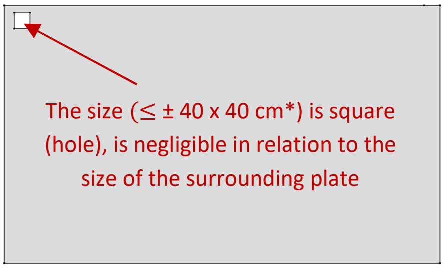
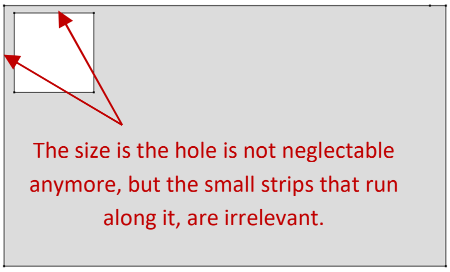
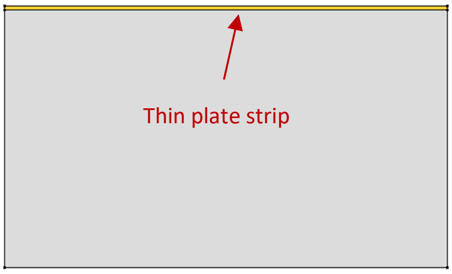
The first step in ‘Design’ is making an analysis model out of the physical model. Although this step is often underestimated, it is of major importance because **garbage in = garbage out**! You may not expect good results from a sloppy analysis model.

Making an analysis model cannot be done without simplifications. Yet some designers have trouble understanding what is relevant for the analysis and what not. Some common mistakes:

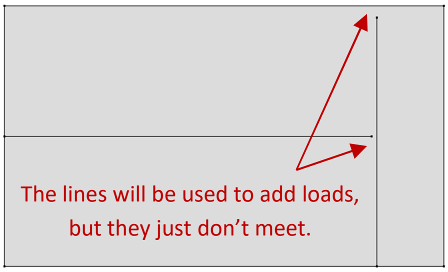
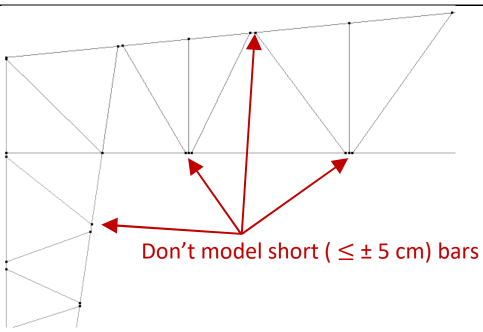
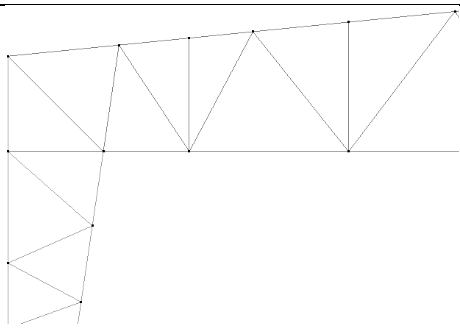
- Points and lines with no (structural) purpose.

WRONG	CORRECT
	
	

- Don’t model small plates/holes.

WRONG	CORRECT
	
	
	

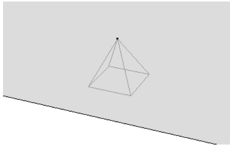
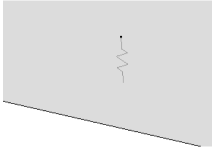
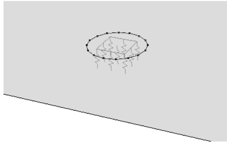
- Join elements close to each other.

WRONG	CORRECT
	
	

* Source: Constructieleer, Gewapend beton 2, p281, ISBN 978-94-6104-006-0

2.2. Method 2: choice of supports

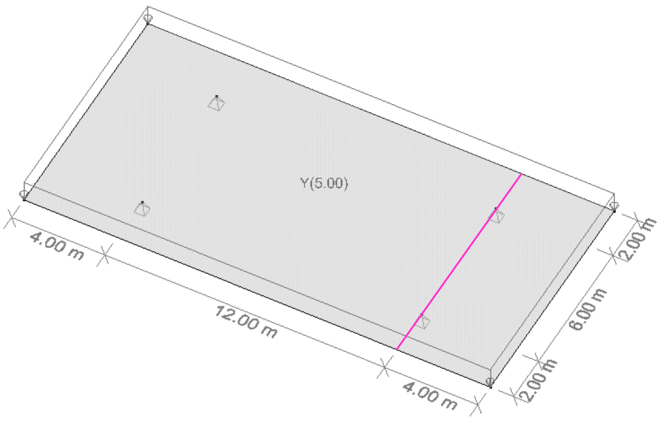
Singularities near point/ line supports arise because the real dimensions of the supports are neglected. As the supports usually have no dimensions in FEM software, the theoretical values will become more present with smaller mesh sizes. The theoretical value is infinite for a point support. As an alternative, springs could be used. An overview:

Fixed support(line)	**1 (or more) elastic point spring(s)**	**Elastic foundation**
		
Quick & simple	Quick & simple	More complex
Only 1 point / line required	Only 1 point / line required	Additional points & plate required
Large mesh possible	Large mesh possible	Small mesh required
No deformation in the support	Realistic deformation	Realistic deformation
Sensitive for peaks	(Limited) damping peak	Damping peak
No redistribution	Redistribution of the forces	Redistribution of the forces
Interesting for large models	Interesting for large models	Interesting for small models
$k = fixed$	$k = \frac{E_{column} \cdot A_{column}}{l_{column}}$	$k = \frac{E_{column}}{l_{column}}$

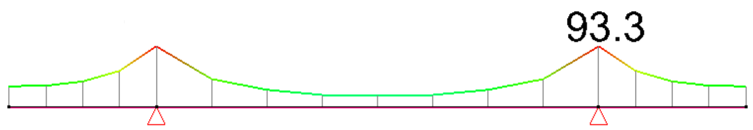
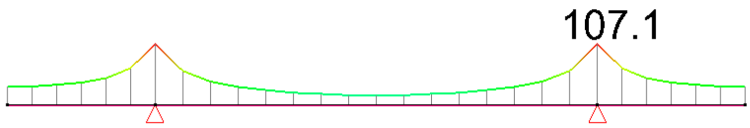
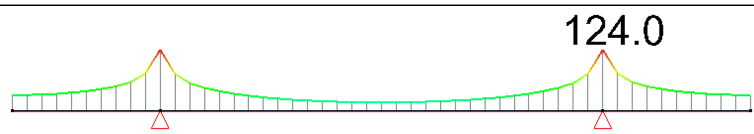
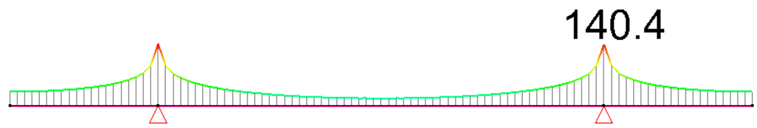
2.3. Method 3: choice of the mesh size

To test which mesh size would make sense, we compare theoretical results with Diamonds results.

Consider a 2 way slab (thickness 20cm) with the following dimensions and supports. The applied load is 5KN/m². A symmetrical mesh is used.



We compare the total moment along the pink cut line in Diamonds with the theoretical value 400kNm (= 5kN/m²*4m*10m*4m/2) for different mesh sizes. In the process, we also compare the peak value of the moment above the supports.

0.8m (4 x plate thickness)		404.9kNm (101.2%) 93.3kNm (100%)
0.4m (2 x plate thickness)		404.6kNm (101.2%) 107.1kNm (114.8%)
0.2m (1 x plate thickness)		406.5kNm (101. 6%) 124kNm (132.9%)
0.1m (0.5 x plate thickness)		408.9kNm (102.2%) 140.4kNm (150.5%)

Conclusions:

- How smaller the mesh, the better the theoretical values are approached. But the higher the peak value above the support (=singularities) and the longer the calculation time.
- You may never compare peak results.
- If you take the maximum mesh size between 2 and 4 times the plate thickness, the obtained results are acceptable within the engineering accuracy.
  Don't choose a mesh size smaller than the thickness of the plates.

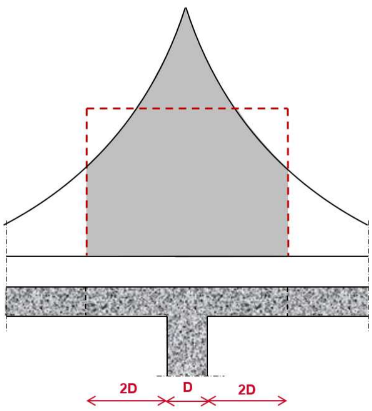
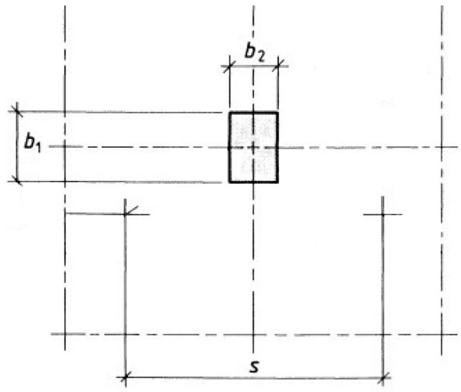
2.4. Method 4: Smear out according to 'Plates and FEM'

This rule will spread out the moment over a distance s[3]:

$$s = 5 \cdot D$$

With:

- D the diameter of the column
  For a rectangular column D can be taken as $min(b_1, b_2)$, $max(b_1, b_2)$ or $0.5(b_1 + b_2)$, … It is up to the engineer to make a responsible choice.

Front view	Top view
	

See §2.6 how this is practically done in Diamonds.

[3] In the article 'Spreiding piekmomenten in vlakke plaatvloeren' you'll find 2d + D, with 'd' the effective height of the slab and D the diameter of the column.

2.5. Method 5: Smear out according to 'NEN 6720'

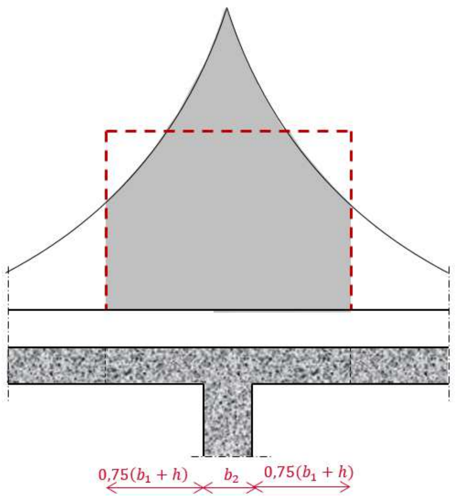
This rule will spread out the moment over a distance s.

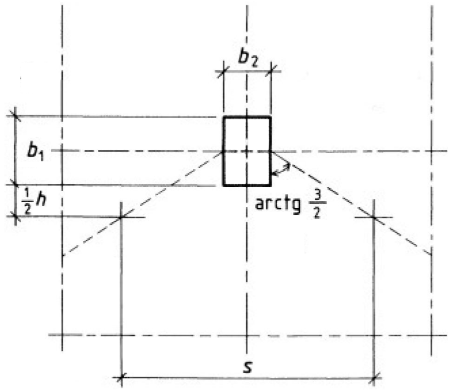
$$s = b_2 + 1.5b_1 + 1.5h$$

With:

- b_1, b_2 the dimensions of the column
- h the thickness of the plate

Front view	Top view





See §2.6 how this is practically done in Diamonds.

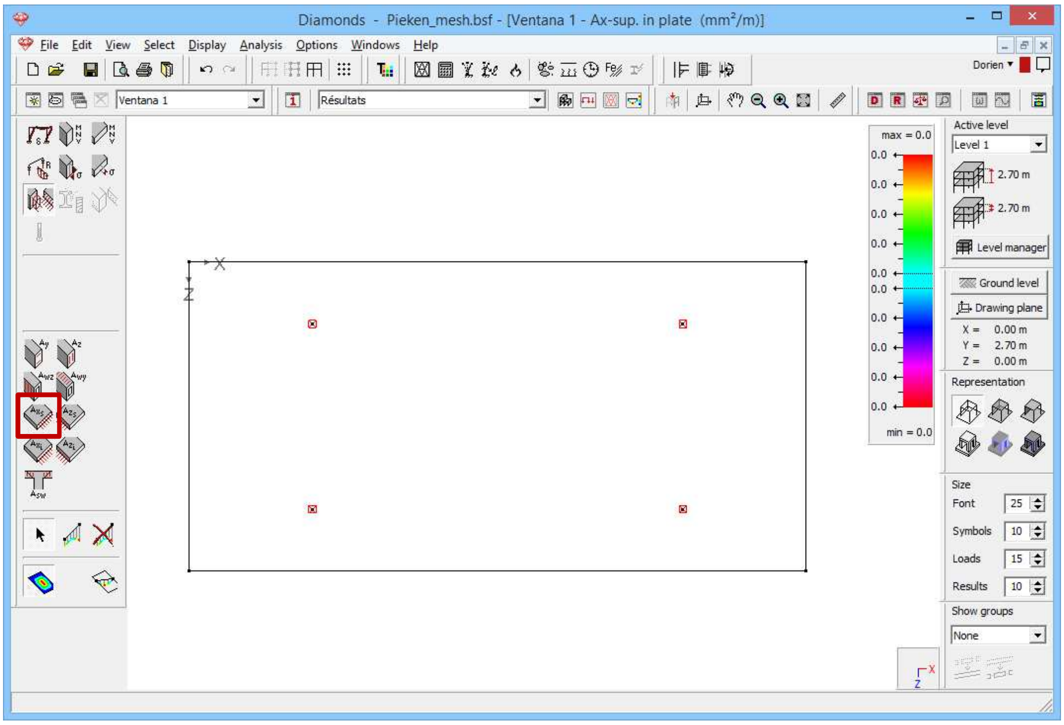
2.6. Practical: smearing out in Diamonds

2.6.1. Using cut lines

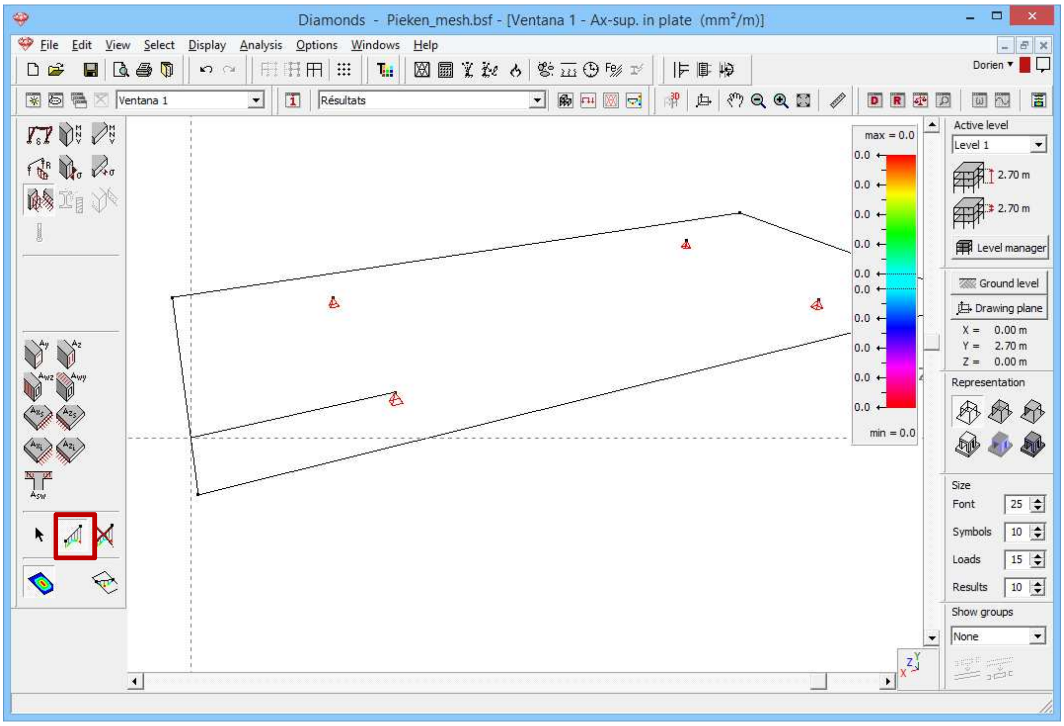
In Diamonds, the spreading rule can be applied using cut lines. The same model as in §2.3 is used[4].

- Temporarily turn off the results by clicking once on the active result icon.

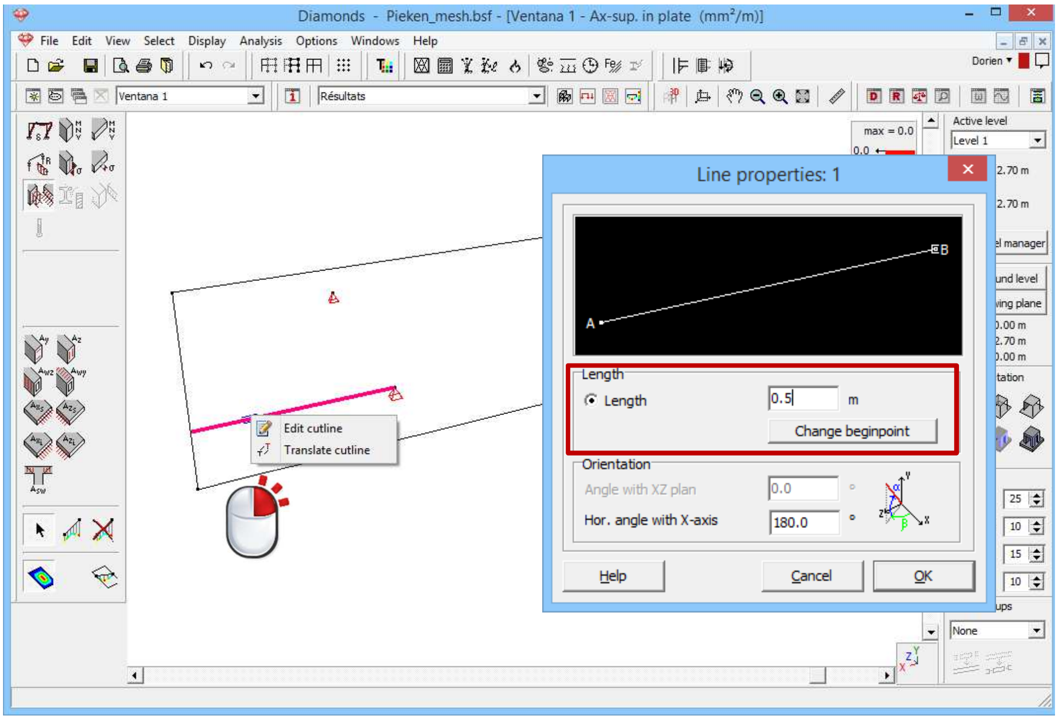
[4] only the reinforcement in ULS is considered.



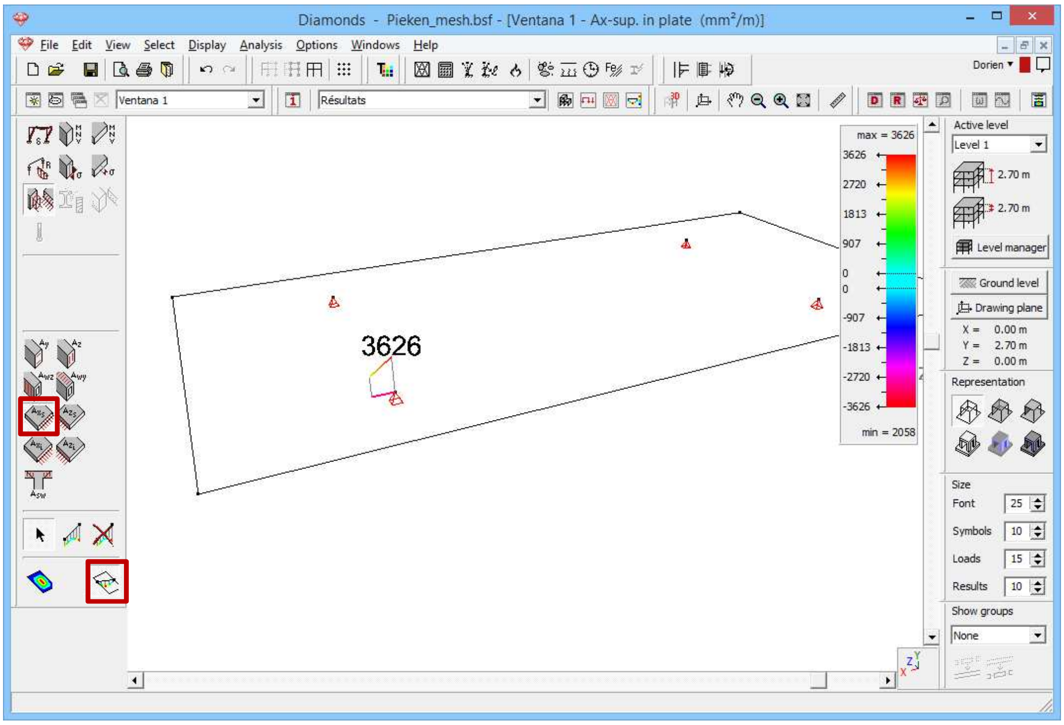
- Draw a cutline from the support to the border of the plate.



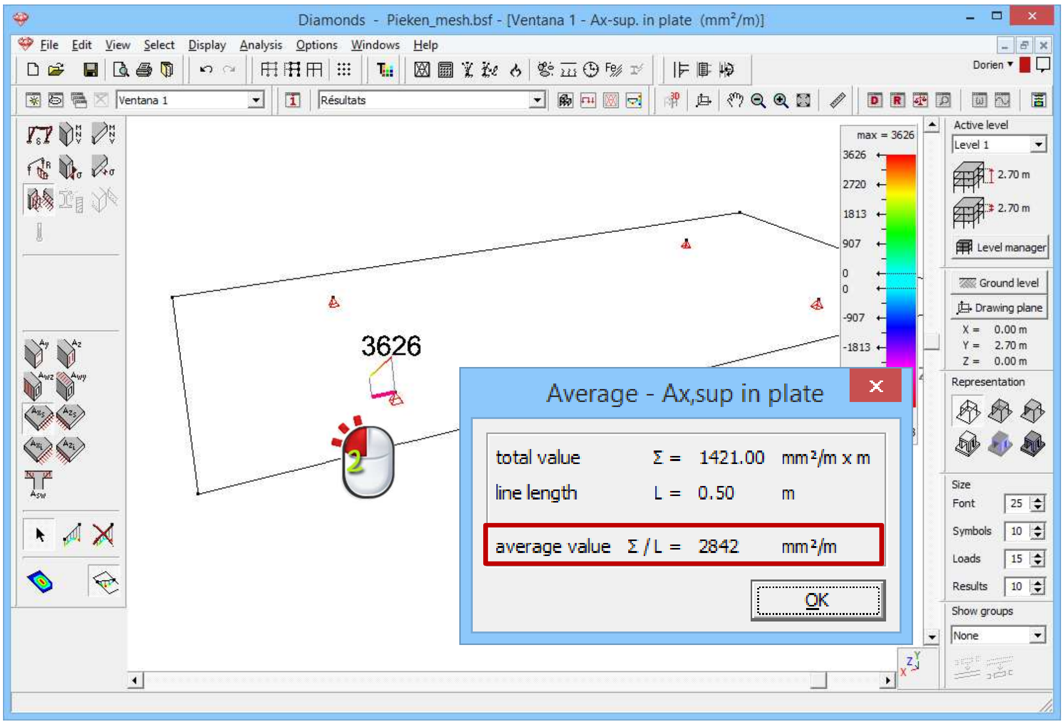
- Modify the length of the cut line to the calculated spreading length (either using §2.4 or §2.5). Suppose a spreading **radius** of 50cm.
  - Switch the begin/end point with Change beginpoint.
  - Enter the desired length.



- Show the results on a cutline  instead of on the entire model. Also show the reinforcement results again.



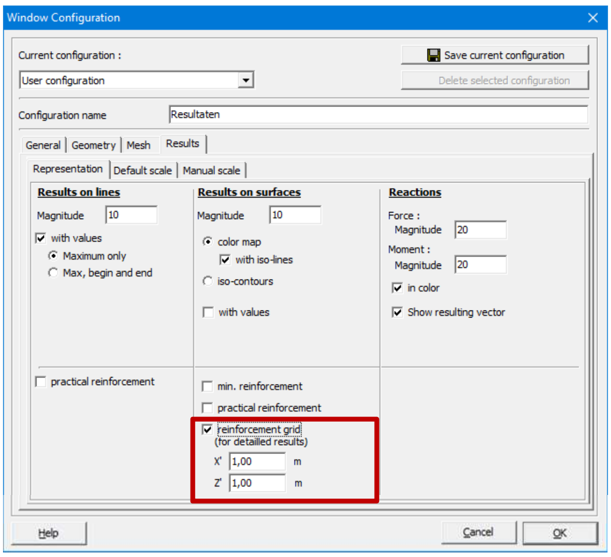
- Double click on the cut line for the total value and mean value. It is the mean value that will be applied in that area, not the peak value.



2.6.2. Using reinforcement grid

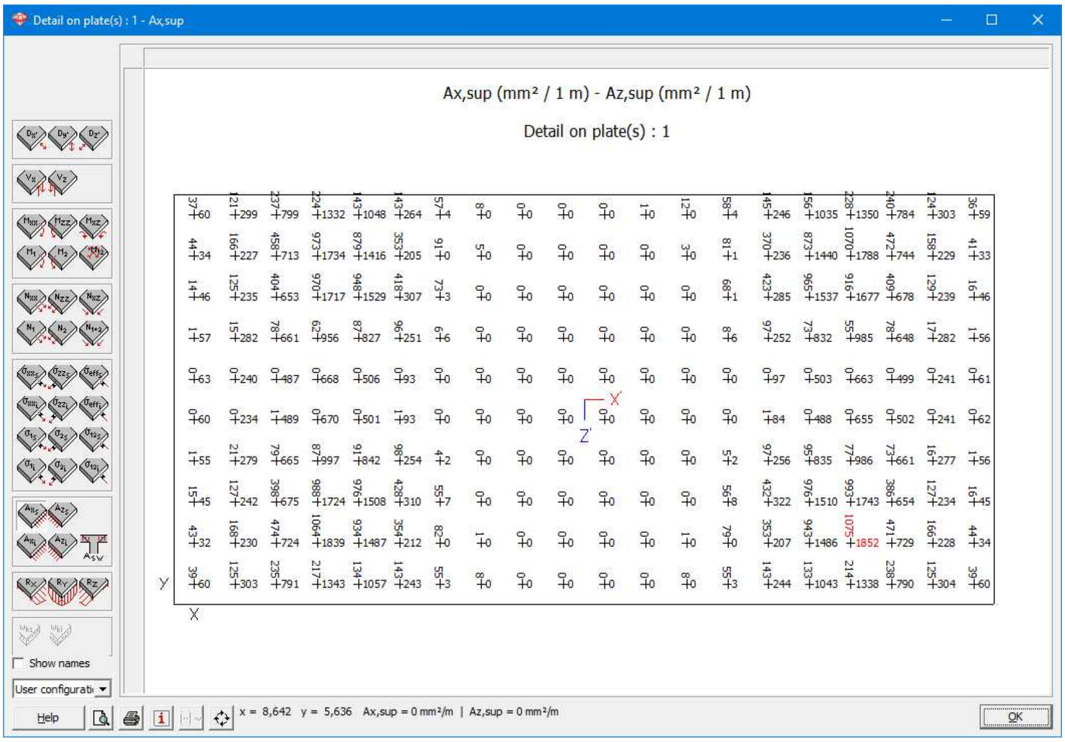
In Diamonds, the spreading rule can be applied using the reinforcement grid. The same model as in §2.3 is used[5].

- Click on . Check the option for the reinforcement grid and set the grid step equal to the calculated (either using §2.4 or §2.5) spreading length. Since we used a spreading **radius** of 50cm in §2.6.1, we're going to use the spreading **diameter** here, so 1m.



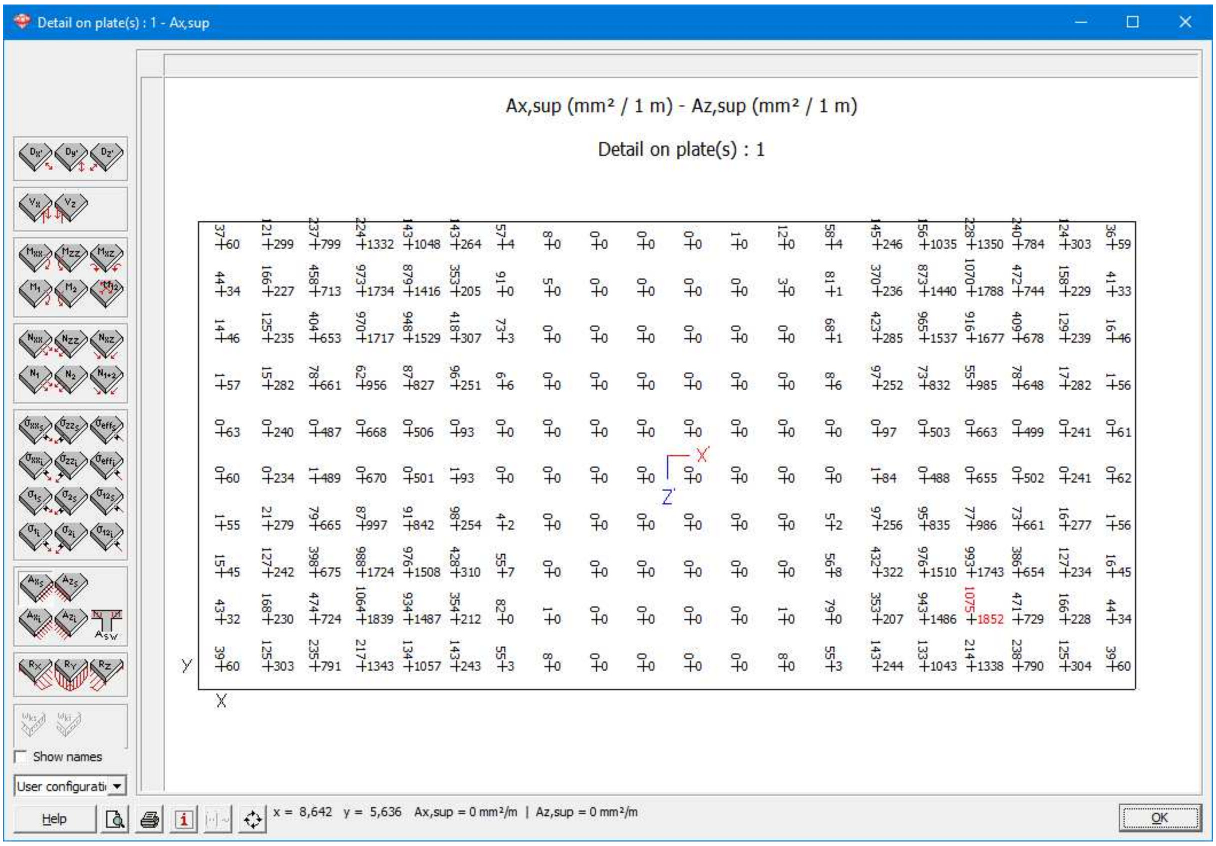
- Show the results for the reinforcement in the plates. Select the plate and click on .
  In grid will calculate the mean reinforcement (in a grid equal to the entered grid step). The mean reinforcement in each direction is given next to the '+' sign. The largest value for the reinforcement in each direction is given in red.

[5] only the reinforcement in ULS is considered.



Notes:

- To change the font size:
  - Close the detailed results
  - Change the font size to the desired value.
  - Select the plate again, click on .
- Since the reinforcement grid calculated the mean value over the grid step, the grid step should be chosen wisely. The dimensions of the plate in example §2.3 are 10 x 20m. If the grid step is set to 10m, the reinforcement would be smeared out over nearly the entire surface of the plate!



3. Conclusions

- Method 1 and 3 are indispensable in good FEM design.
- It is up to the engineer to make a judicious decision in the use of Methods 2, 4, 5 or a combination of these methods.
- Method 5 will not always result in a more conservative approach than Method 4.

4. Sources

[1] J. Blaauwendraad, Plates and FEM – Surprises and Pitfalls - §11.2 and §14, Springer Dordrecht, 2010, ISBN978-90-481-3595-0

[2] NEN 6720, Voorschriften beton – Constructieve eisen en rekenmethoden (VBC 1995) §7.5.3.4

[3] EN 1992-1-1:2005, Eurocode 2: Ontwerp en berekening van betonconstructies – Deel 1-1: Algemene regels en regels voor gebouwen (+AC:2010)