Design (timber)

1. Introduction

To assess a structure (whether it is sufficient or not), the following steps must be followed in sequence:

- Make a mechanics model of the structure and loads
- Calculate the internal forces and moments (= global analysis)
- **Verification of the cross sections (= design)**
    - o A check in the ultimate limit state UGT (§2**Fout! Verwijzingsbron niet gevonden.**2.2)
        - Strength (will the structure break?)
        - Stability (will the structure buckling or undergo lateral torsional buckling?)
    - o A check in the service limit state SLS (§32.3)
        - The deformations may not be too high.
        - The vibrations must remain within the limits.

This document provides background in the design of timber.

Suggestions / additions to this document are always welcome at op info@buildsoft.eu.

2. Verification in ULS

2.1. Material properties

The most special mechanical properties of steel are:

- Characteristic bending strength $f_{m,k}$ $[N/mm^2]$
- Characteristic compressive strength along the grain $f_{c,0,k}$ $[N/mm^2]$
- Characteristic panel shear strength $f_{v,k}$ $[N/mm^2]$
- Young's modulus $E_{0,gem}$ $[kN/mm^2]$
- Mean value of shear modulus G_{mean} $[N/mm^2]$
- Characteristic density $\rho_k [kg/m^3]$

Product standards like EN 388 give an overview of the normalized timber qualities with accompanying properties.

Characteristic values for the strength properties of solid timber (according to the product norm EN 338)

SOLID TIMBER		C14	C16	C18	C20	C22	C24	C27	C30	C35	C40	C45	C50
$f_{m,k}$	$[N/mm^2]$	14	16	18	20	22	24	27	30	35	40	45	50
$f_{t,0,k}$	$[N/mm^2]$	8	10	11	12	13	14	16	18	21	24	27	30
$f_{c,0,k}$	$[N/mm^2]$	16	17	18	19	20	21	22	23	25	26	27	29
$f_{v,k}$	$[N/mm^2]$	3,0	3,2	3,4	3,6	3,8	4,0	4,0	4,0	4,0	4,0	4,0	4,0
$E_{0,mean}$	$[kN/mm^2]$	7	8	9	9,5	10	11	11.5	12	13	14	15	16
G_{mean}	$[kN/mm^2]$	0,44	0,50	0,56	0,59	0,63	0,69	0,72	0,75	0,81	0,88	0,94	1,00
ρ_k	$[kg/m^3]$	290	310	320	330	340	350	370	380	400	420	440	460

The properties in the table above are characteristic values. To make it design values, they are multiplied by a factor k_{mod} and divided by a partial safety coefficient γ_M.

$$X_d = k_{mod} \frac{X_k}{\gamma_M}$$

2.1.1. Partial safety coefficient γ_M

Recommended partial factors γ_M for material properties and resistances (EN 1995-1-1 §2.3)

Solid timber	1.30
Glued lamented timber	1.25
LVL, plywood, OSB	1.20

2.1.2. Modification factor k_{mod}

Duration of load and moisture content affect the strength and stiffness properties of timber and wood-based elements. The effect on the strength properties is taken into account by multiplying them with the modification factor k_{mod}.

k_{mod} depends on:

- The type of timber
- The service class
- The load-duration class

Service class as a function of the relative humidity (EN 1995-1-1 §2.3.1.3)

Service class	Relative humidity	Moisture content φ
1	exceeding 65 % for a few weeks per year	$\varphi \leq 12\%$
2	exceeding 85 % for a few weeks per year	$12\% < \varphi \leq 20\%$
3	exceeding the limits of service class 2	$\varphi > 20\%$

Values for the modification factor k_{mod} (EN 1995-1-1 Table 3.1)

Material	Climate class	Load duration classes				
		Permanent	Long-term	Medium-term	Short-term	Instantaneous
Solid timber (C and D)	1	0.60	0.70	0.80	0.90	1.10
	2	0.60	0.70	0.80	0.90	1.10
	3	0.50	0.55	0.65	0.70	0.90
Glued laminated timber (GL)	1	0.60	0.70	0.80	0.90	1.10
	2	0.60	0.70	0.80	0.90	1.10
	3	0.50	0.55	0.65	0.70	0.90
LVL	1	0.60	0.70	0.80	0.90	1.10
	2	0.60	0.70	0.80	0.90	1.10
	3	0.50	0.55	0.65	0.70	0.90

2.2. Strength check

Timber check is possible for the following shapes: ■, █ (constant and variable), ●, **T**.

- EN 1995-1-1 §6.1.2 Tension (parallel to the fibre direction)

$$\frac{N_{Ed}}{N_{t,Rd}} \leq 1{,}0$$
$$N_{t,Rd} = A \cdot f_{t,0,d}$$

- EN 1995-1-1 §6.1.4 Compression (parallel to the fibre direction)

$$\frac{N_{Ed}}{N_{c,Rd}} \leq 1{,}0$$
$$N_{c,Rd} = A \cdot f_{c,0,d}$$

- EN 1995-1-1 §6.2.5 and §6.4.2 Bending

$$max(\frac{M_{y',Ed}}{k_{m,\alpha} \cdot M_{y',Rd}} + k_m \frac{M_{z',Ed}}{M_{z',Rd}}, k_m \frac{M_{y',Ed}}{k_{m,\alpha} \cdot M_{y',Rd}} + \frac{M_{z',Ed}}{M_{z',Rd}}) \leq 1$$
$$M_{y',Rd} = W_{el,y'} \cdot f_{m,d}$$
$$M_{z',Rd} = W_{el,z'} \cdot f_{m,d}$$

With:

- o k_m makes allowance for re-distribution of stresses and the effect of inhomogeneity's of the material in a cross-section.

Timber type	Type of cross section	k_m
solid timber, glued laminated timber and LVL	█	0,7
	all other	1,0
other wood-based structural products		1,0

- o $k_{m,\alpha}$ takes into account the influence of the changing height of variable sections on the bending stresses parallel to the surface.

1	For constant sections
$\frac{1}{\sqrt{1+\left(\frac{f_{m,d}\tan\alpha}{0.75 f_{v,d}}\right)^2+\left(\frac{f_{m,d}\tan^2\alpha}{f_{t,90,d}}\right)^2}}$	For tensile stresses parallel to the tapered edge
$\frac{1}{\sqrt{1+\left(\frac{f_{m,d}\tan\alpha}{1.50 f_{v,d}}\right)^2+\left(\frac{f_{m,d}\tan^2\alpha}{f_{c,90,d}}\right)^2}}$	For compressive stresses parallel to the tapered edge

- o $N_{t,Rd}$ see above
- o $M_{y',Rd}$, $M_{z',Rd}$ and k_m see above

- EN 1995-1-1 §6.1.7 and EN 1995-1-1:2004/A1:2008 §6.1.7 (2) Shear force

$$\frac{V_{Ed}}{V_{i,Rd}} \leq 1{,}0$$

$$V_{i',Rd} = \frac{2}{3}A_{ef} \cdot f_{v,d} = \frac{2}{3}f_{v,d} \cdot b_{ef} \cdot h = \frac{2}{3}f_{v,d} \cdot \underbrace{k_{cr} \cdot b}_{b_{ef}} \cdot h$$

The index i is the axis according to which the shear force is considered.

Timber type	k_{cr}
Solid/ glued timber	0.67
Other	1.00

- EN 1995-1-1 §6.1.8 Torsion

$$\frac{T_{Ed}}{k_{shape} \cdot T_{Rd}} \leq 1{,}0$$
$$T_{Rd} = T_{wm} \cdot f_{v,d}$$

Cross section[1]	$T_{wm,el}$	k_{shape}[2]
■ $a = min(b, h)$	$0.208a^2$ if $a = b$	$min\begin{cases}1 + 0.05\frac{h}{b} \\ 1{,}2\end{cases}$
	$\frac{a^2b^2}{3b + 1.8a}$ if $b < 10a$	
	$\frac{a^2b}{3}$ if $b > 10a$	
●	$\frac{\pi H^3}{16}$	1,2

- EN 1995-1-1 §6.2.3 Bending and tension

$$max(\frac{N_{Ed}}{N_{t,Rd}} + \frac{M_{y',Ed}}{k_{m,\alpha} \cdot M_{y',Rd}} + k_m \frac{M_{z',Ed}}{M_{z',Rd}}, \frac{N_{Ed}}{N_{t,Rd}} + k_m \frac{M_{y',Ed}}{k_{m,\alpha} \cdot M_{y',Rd}} + \frac{M_{z',Ed}}{M_{z',Rd}}) \leq 1$$

With:
- o $N_{t,Rd}, M_{y',Rd}, M_{z',Rd}, k_{m,\alpha}$ and k_m see above

- EN 1995-1-1 §6.2.4 Bending and compression

$$max(\left(\frac{N_{Ed}}{N_{c,Rd}}\right)^2 + \frac{M_{y',Ed}}{k_{m,\alpha} \cdot M_{y',Rd}} + k_m \frac{M_{z',Ed}}{M_{z',Rd}}, \left(\frac{N_{Ed}}{N_{c,Rd}}\right)^2 + k_m \frac{M_{y',Ed}}{k_{m,\alpha} \cdot M_{y',Rd}} + \frac{M_{z',Ed}}{M_{z',Rd}}) \leq 1$$

With:
- o $N_{c,Rd}, M_{y',Rd}, M_{z',Rd}, k_{m,\alpha}$ and k_m see above

2.3. Stability check

- EN 1995-1-1 §6.3.2 Buckling and interaction

$$\lambda_y = \frac{L_{cr.y}}{i_y}, \lambda_z = \frac{L_{cr.z}}{i_z}, \overline{\lambda_y} = \frac{\lambda_y}{\pi}\sqrt{\frac{f_{c,0,k}}{E_{0,05}}}, \overline{\lambda_z} = \frac{\lambda_z}{\pi}\sqrt{\frac{f_{c,0,k}}{E_{0,05}}}$$

$(\overline{\lambda_y}$ and $\overline{\lambda_z}) \leq 0.3$	$else$
$\left(\frac{N_{c,Ed}}{N_{c,Rd}}\right)^2 + \frac{M_{y',Ed}}{k_{m,\alpha} \cdot M_{y',Rd}} + k_m \frac{M_{z',Ed}}{M_{z',Rd}} \leq 1$	$\frac{N_{c,Ed}}{k_{cr,y}N_{c,Rd}} + \frac{M_{y',Ed}}{k_{m,\alpha} \cdot M_{y',Rd}} + k_m \frac{M_{z',Ed}}{M_{z',Rd}} \leq 1$
EN	EN
$\left(\frac{N_{c,Ed}}{N_{c,Rd}}\right)^2 + k_m \frac{M_{y',Ed}}{k_{m,\alpha} \cdot M_{y',Rd}} + \frac{M_{z',Ed}}{M_{z',Rd}} \leq 1$	$\frac{N_{c,Ed}}{k_{cr,z}N_{c,Rd}} + k_m \frac{M_{y',Ed}}{k_{m,\alpha} \cdot M_{y',Rd}} + \frac{M_{z',Ed}}{M_{z',Rd}} \leq 1$

With:
- o $N_{c,Rd}, M_{y',Rd}, M_{z',Rd}, k_{m,\alpha}$ and k_m see above
- o $L_{cr,i}$ is the relevant buckling length
- o $k_{cr,y}$ and $k_{cr,z}$ see EN 1995-1-1 §6.3.2

[1] Berekening van constructies, deel I, Van de pitte, p138
[2] EN 1995-1-1/A2: June 2014

- EN 1995-1-1 §6.3.3 Lateral torsional buckling

$$\frac{M_{y',Ed}}{k_{crit}k_{m,\alpha}M_{y',Rd}} \leq 1$$

$$k_{crit} = \begin{cases} 1 & if\ \lambda_{rel.m} \leq 0.75 \\ 1.56 - 0.75\lambda_{rel.m} & if\ 0.75 < \lambda_{rel.m} \leq 1.4 \\ \frac{1}{\lambda_{rel.m}^2} & \lambda_{rel.m} > 1.4 \end{cases}$$

$$\lambda_{rel.m}^2 = \sqrt{\frac{f_{m.k}}{\sigma_{m.crit}}} = \sqrt{\frac{f_{m.k}}{\frac{M_{y.crit}}{W_{y.el}}}} = \sqrt{\frac{f_{m.k}}{\frac{\pi\sqrt{E_{0.05}I_zG_{0.05}I_t}}{l_{ef}W_{y.el}}}}$$

With:
- $N_{c,Rd}, M_{y',Rd}, M_{z',Rd}, k_{m,\alpha}$ and k_m see above
- k_{crit} see EN 1995-1-1 §6.3.3
- l_{ef} is the effective length of the beam, depending on the support conditions and the load configuration, see EN 1995-1-1 Table 6.1
- I_z is the second moment of area about the weak axis z
- I_t is the torsional moment of inertia

- EN 1995-1-1 §6.3.3 Interaction

$$\left(\frac{M_{y',Ed}}{k_{crit}k_{m,\alpha}M_{y',Rd}}\right)^2 + \frac{N_{c,Ed}}{k_{cr,z}N_{c,Rd}} \leq 1$$

With:
- $N_{c,Rd}, M_{y',Rd}, k_{m,\alpha}, k_{crit}$ and k_m see above
- $k_{cr,y}$ and $k_{cr,z}$ see EN 1995-1-1 §6.3.2

3. Verification in SLS

Duration of load and moisture content affect the strength and **stiffness properties** of timber and wood-based elements. The effect on the **stiffness** properties is taken into account by a modification factor k_{def}.

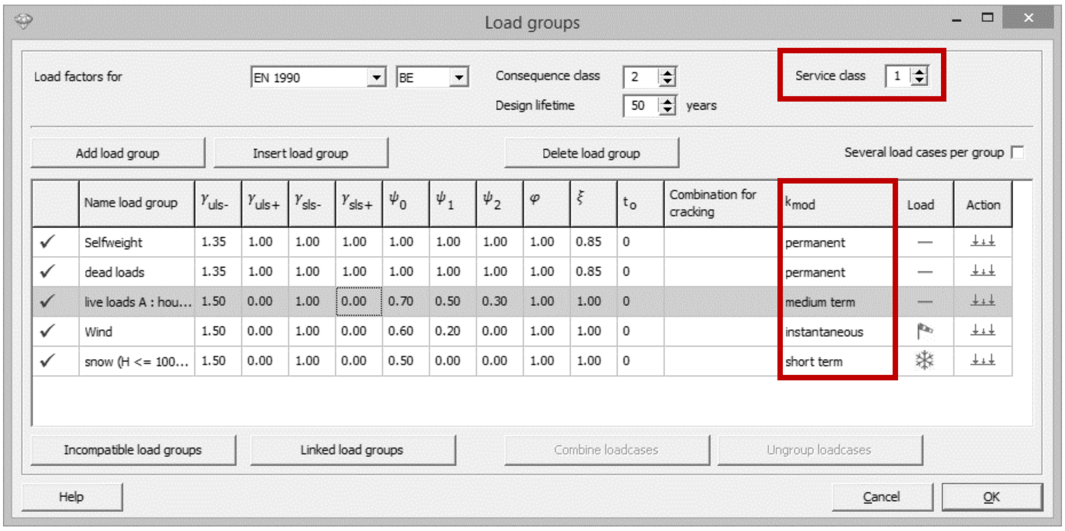
Values for the modification factor k_{def} (EN 1995-1-1 Table 3.2)

Material	Climate class		
	1	2	3
Solid timber (C and D)	0.60	0.80	2.00
Glued laminated timber (GL)	0.60	0.80	2.00
LVL	0.60	0.80	2.00

Diamonds calculates the displacement, deformations and vibrations, but it is up to user to evaluate them since the limits are project dependent.

4. Implementation in Diamonds

- The effect of member size on the strength (EN 1995-1-1 §3.2(3), §3.3(3), §3.4(3)) is not taken into account in Diamonds.
- The values for k_{mod} and k_{def} can be consulted in the material library (Edit > Material library).
- To determine the correct value for k_{mod} and k_{def}, the user must define the correct duration for each load group and the service class.
  If a combination contains loads with a different duration, then Diamonds automatically will apply §3.1.3 (2).



- To take EN 1995 §3.2 (4) into account, you'll need to:
  - Open the material library.
  - Make a copy of the material.
  - Adjust k_{def}.
  - Close material library and save changes.
  - Assign the copied material to the structure.
- The checks mentioned in §2.2 and §2.3 are carried out by Diamonds.
- The effective length l_{ef} equals $\frac{L}{C_1} + 2h$ in Diamonds.
  $\frac{L}{C_1}$ because EN 1995-1-1 only gives formula for l_{ef} for some cases.
  C_1 is a factor which takes the moment distribution into account. Determined according to Equivalent uniform moment factors for lateral- torsional buckling handling or steel members - Journal of constructional Steel Research No 62, 2006; Serna, Lopez, Puente and Young.
- The results of the strength and stability check are displayed as a percentage of the total capacity (= 100%) of the section.
  - A structure is sufficient if both for the strength and stability checks give percentages less than 100%.
  - A structure is NOT sufficient if the strength and/or stability check give percentages more than 100%.

  Based on the percentage no judgment can be made on the extent to which a section is sufficient or not. This is because the formulas used §2.2 and §2.3 are not always linear.

- The intermediate results can be retrieved by double-clicking a bar while the results for the strength or stability verification are displayed.