Design (steel)

1. Introduction

To assess a structure (whether it is sufficient or not), the following steps must be followed in sequence:

- Make a mechanics model of the structure and loads
- Calculate the internal forces and moments (= global analysis)
- **Verification of the cross sections (= design)**
  - A check in the ultimate limit state UGT (§22.3)
    - Strength (will the structure break?)
    - Stability (will the structure buckle or undergo lateral torsional buckling?)
  - A check in the service limit state SLS (§3)
    - The deformations may not be too high.
    - The vibrations must remain within the limits.

This document provides background in the design of steel.

Suggestions / additions to this document are always welcome at op info@buildsoft.eu.

2. Verification in ULS

2.1. Material properties

The most special mechanical properties of steel are:

- Young's modulus E
- The yielding strength f_y
- The ultimate tensile strength f_u

These properties can be determined using destructive tensile test.

EN 1993-1-1 Table 3.1 gives an overview of the normalized steel grades with accompanying yielding and ultimate tensile strength. The national annexes may deviate from this.

Nominalized values for the yielding f_y and ultimate tensile strength f_u for hot rolled structural steel (according to the product norm EN 10025-2)

Steel quality	thickness t in mm			
	$\leq 40\ mm$		$40\ mm - 80\ mm$	
	f_y (N/mm²)	f_u (N/mm²)	f_y (N/mm²)	f_u (N/mm²)
S 235	235	360	215	360
S 275	275	430	255	410
S 355	355	510	335	470
S 450	440	550	410	550

In practice, the material exhibits imperfections such as residual stresses. The thicker the material, the greater the chance of residual stress, hence the decreasing yield strength f_y.

2.2. Cross section properties

The properties (the area A and the resistance moment W to be specific) of profiles subjected to compression and bending highly depend on the ratio c/t between the width c and the thickness t of the compressed parts (flange or web) of the cross-section. The smaller the ratio c/t, the slimmer the compressed part, the smaller the bearing capacity. The bearing capacity for slim profiles is limited by local buckling of the compressed parts.

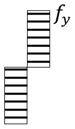
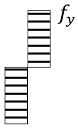
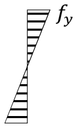
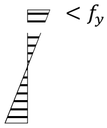
Examples of local buckling[1]

Local buckling of the web

Local buckling of the flanges

Therefore, sections are divided into 4 classes based on the resistance moment W that can develop in the cross section.

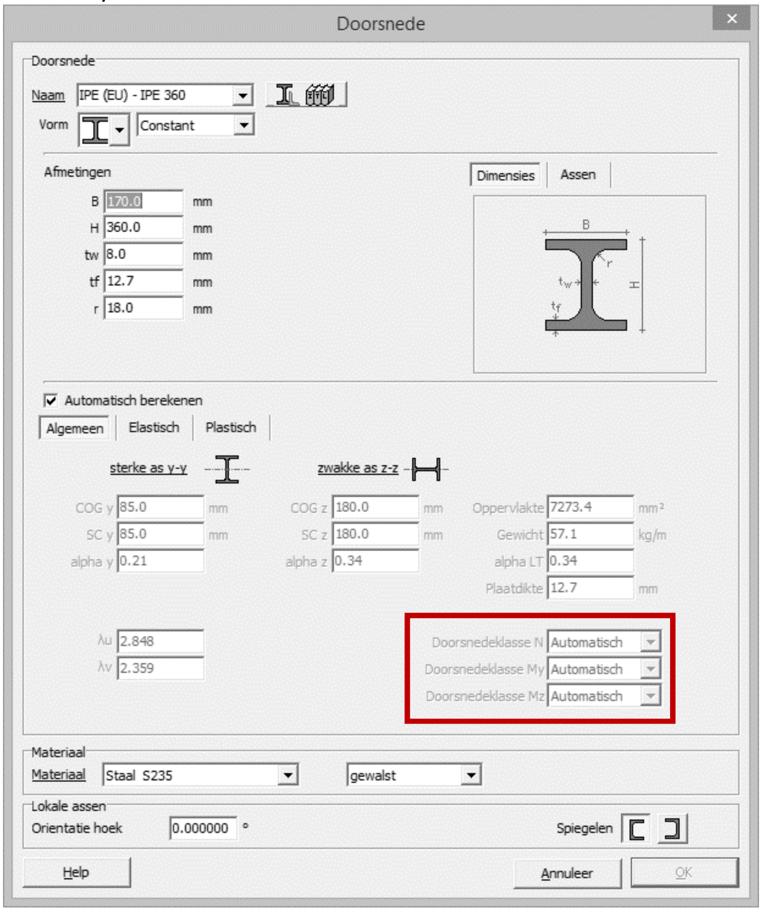
[1] https://ceprofs.civil.tamu.edu/llowery/cven446/Syllabi/446_18a_42days.htm

	Class 1 Plastic	**Class 2** Compact	**Class 3** Semi-compact	**Class 4** Slender
Stress distribution				
Global analysis[2]	Plastic[3]	Elastic	Elastic	Elastic
Steel verification	The yielding strength can be reached in the entire cross section. Plastic section properties are used in the design.	The yielding strength can be reached in the entire cross section. Plastic section properties are used in the design.	The yielding strength can only be reached in the extreme fibre. Elastic section properties are used in the design.	Local buckling prevents the yielding strength to be reached. Reduced (syn. effective) section properties are used in the design.

The determination to which class a profile belongs to is based on the c/t-ratio of the compressed components and the steel grade (EN 1993-1-1 Table 5.2). The class is determined for the flange and for the web. The class of the section is the most unfavourable of both. If a section does not belong to class 3, then it is of class 4.

Implementation in Diamonds:

- Both axial force, bending moment M_y and bending moment M_z can cause compressive stresses, therefor you can define a class for each of them.



- Diamonds will determine automatically the class for profiles from the standard library  and the default shapes . For user defined sections  it is up to the user to impose the classes. If the profile is off class 4, he/ she should also impose the effective properties.

[2] Informative because the global analysis in all BuildSoft software is elastic.
[3] Only profiles of class 1 possess enough rotational capacity to form a plastic hinge.

2.3. Strength check

The index $'i'$ is

- the axis according to which the shear force is considered,
- the axis around which the moment is considered,
- the axis around which buckling is considered.

For double symmetric profiles this will be the local y' and z' axis. For all other the principle axes of inertia u and v are used.

- EN 1993-1-1 §6.2.3 Tension

$$\frac{N_{Ed}}{N_{t,Rd}} \leq 1{,}0$$
$$N_{t,Rd} = N_{pl,Rd} = A \cdot f_y$$

- EN 1993-1-1 §6.2.4 Compression

$$\frac{N_{Ed}}{N_{c,Rd}} \leq 1{,}0$$

Class 1, 2 and 3	Class 4
$N_{c,Rd} = A \cdot f_y$	$N_{c,Rd} = A_{eff} \cdot f_y$

- EN 1993-1-1 §6.2.5 Bending

$$\frac{M_{Ed}}{M_{i,Rd}} \leq 1{,}0$$

Class 1 and 2	Class 3	Class 4
$M_{i,Rd} = W_{i,pl} \cdot f_y$	$M_{i,Rd} = W_{i,el} \cdot f_y$	$M_{i,Rd} = W_{i,eff} \cdot f_y$

- EN 1993-1-1 §6.2.6 Shear force

$$\frac{V_{Ed}}{V_{i,pl,Rd}} \leq 1{,}0$$
$$V_{i,pl,Rd} = A_{i,v} \cdot \left(f_y/\sqrt{3}\right)$$

Cross section (EN 1993-1-1 §6.2.6 (3))	**Shear area $A_{i,v}$**
Rolled I and H sections, load parallel to web	$A - 2bt_f + (t_w + 2r)t_f \geq h_w t_w$
Rolled C-sections, load parallel to web	$A - 2bt_f + (t_w + r)t_f$
Rolled T-sections, load parallel to web	$0.9\left(A - bt_f\right)$
Welded I, H and box sections, load parallel to web	$\eta \sum (h_w t_w)$
Welded I, H, C and box sections, load parallel to flanges	$A - \sum (h_w t_w)$
Rolled rectangular hollow sections of uniform thickness • Load parallel to depth • Load parallel to width	 $\frac{A \cdot h}{b + h}$ $\frac{A \cdot b}{b + h}$
Circular hollow sections and tubes of uniform thickness	$\frac{2A}{\pi}$

- EN 1993-1-1 §6.2.7 Torsion

$$\frac{T_{Ed}}{T_{Rd}} \leq 1{,}0$$

Class 1 and 2	Class 3	Class 4
$T_{Rd} = T_{wm,pl} \cdot f_{yd}/\sqrt{3}$	$T_{Rd} = T_{wm,el} \cdot f_{yd}/\sqrt{3}$	$T_{Rd} = T_{wm,eff} \cdot f_{yd}/\sqrt{3}$

Cross section[4]	$T_{wm,el}$	Remarks
■	$0.208a^2 \; if \; a = b$	$a = min(B, H)$
	$\frac{a^2b^2}{3b+1.8a} \; if \; b < 10a$	
	$\frac{a^2b}{3} \; if \; b > 10a$	
T	$\frac{h_p^3 b_p + B^3(H - h_p)}{3a_m}$	$a_m = max(h_p, B)$
I	$1.3\frac{2t_f^3 B + t_w^3(H - 2t_f)}{3a_m}$	$a_m = max(t_w, t_f)$
L	$\frac{t^3 B + t^3(H - t)}{3t}$	-
[	$\frac{2t_f^3 B + t_w^3(H - 2t_f)}{3a_m}$	$a_m = max(t_w, t_f)$
	$2H(H - t_f)(B - t_w)t$	$t = min(t_w, t_f)$
●	$\frac{\pi H^3}{16}$	-
○	$\frac{\pi(r^4 - (r - t_w)^4)}{2r}$	$r = 0.5H$

- EN 1993-1-1 §6.2.8 Bending and shear force

$$\frac{M_{i,Ed}}{M_{i,V,Rd}} \leq 1{,}0$$

$$M_{i,V,Rd} = (1 - \rho_{i'})M_{i,Rd}$$

$$\text{If } \frac{V_{i',Ed}}{V_{i',pl,Rd}} \leq 0.5 \text{ then } \rho_{i'} = 0 \text{ , else } \rho_{i'} = \left(\frac{2 \cdot V_{i',Ed}}{V_{i',pl,Rd}} - 1\right)^2$$

In case of an I-profile with identical flanges $M_{y,V,Rd} = min\left(\left[W_{y,pl} - \frac{\rho_z \cdot (h_w \cdot t_w)^2}{4 \cdot t_w}\right] \cdot f_{yd}, M_{y,Rd}\right)$.

Class 1 and 2	Class 3	Class 4
$M_{i,Rd} = W_{i,pl} \cdot f_y$	$M_{i,Rd} = W_{i,el} \cdot f_y$	$M_{i,Rd} = W_{i,eff} \cdot f_y$

- EN 1993-1-1 §6.2.9 Bending and axial force

Class 1 and 2

$$\left(\frac{M_{y,Ed}}{M_{N,y,Rd}}\right)^{\alpha} + \left(\frac{M_{z,Ed}}{M_{N,z,Rd}}\right)^{\beta} \leq 1{,}0$$

With:

- For H-sections
  - $M_{N,y,Rd} = M_{y,Rd}$ if $N_{Ed} \leq 0{,}25 \cdot N_{pl}$ and $N_{Ed} \leq 0{,}5 \cdot h_w \cdot t_w \cdot f_{yd}$
  - $M_{N,z,Rd} = M_{z,Rd}$ if $N_{Ed} \leq h_w \cdot t_w \cdot f_{yd}$
  - $\alpha = 2$ and $\beta = max\{1; 5n\}$
- For - sections
  - $M_{N,y,Rd} = min\left\{M_{pl,y,Rd}; \frac{M_{pl,y,Rd}(1-n)}{(1-0.5a)}\right\}$
  - $M_{N,z,Rd} = M_{pl,z,Rd}\left(1 - \left(\frac{n-a}{1-a}\right)^2\right)$ if $n > a$, else $M_{N,z,Rd} = M_{pl,z,Rd}$
  - $n = \frac{N_{Ed}}{N_{pl,Rd}}, a = min\left\{0{,}5 \; ; \; \frac{A - 2bt_f}{A}\right\}$
  - $\alpha = \beta = min\left\{\frac{1{,}66}{1 - 1{,}13n^2}; 6\right\}$

Class 3 and 4

$$\frac{N_{Ed}}{N_{Rd}} + \frac{M_{y,Ed} + \Delta M_{y,Ed}}{M_{y,Rd}} + \frac{M_{z,Ed} + \Delta M_{z,Ed}}{M_{z,Rd}} \leq 1.0$$

With:

- N_{Rd} determined by pure tension or compression
- $M_{y,Rd}$ en $M_{z,Rd}$ determined at the check 'bending'
- If class = 3, then $\Delta M_{y,Ed} = \Delta M_{z,Ed} = 0$, else $\Delta M_{y,Ed} = e_{N,y}N_{Ed}$ and $\Delta M_{z,Ed} = e_{N,z}N_{Ed}$

[4] 'Berekening van constructies', deel I, Van de pitte, p138

- EN 1993-1-1 §6.2.10 Bending, shear and axial force

<table>
<tr><th>Class 1 and 2</th><th>Class 3 and 4</th></tr>
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$$\left(\frac{M_{y,Ed}}{M_{NV,y,Rd}}\right)^{\alpha}+\left(\frac{M_{z,Ed}}{M_{NV,z,Rd}}\right)^{\beta}\leq 1,0$$
$$M_{NV,y,Rd}=(1-\rho_z)M_{N,y,Rd}$$
$$M_{NV,z,Rd}=(1-\rho_y)M_{N,z,Rd}$$

With:

- If $\frac{V_{z,Ed}}{V_{z,pl,Rd}}\leq 0.5$ then $\rho_z=0$, otherwise $\rho_z=\left(\frac{2\cdot V_{z,Ed}}{V_{z,pl,Rd}}-1\right)^2$
- If $\frac{V_{y,Ed}}{V_{y,pl,Rd}}\leq 0.5$ then $\rho_y=0$, otherwise $\rho_y=\left(\frac{2\cdot V_{y,Ed}}{V_{y,pl,Rd}}-1\right)^2$

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$$\frac{N_{Ed}}{N_{Rd}}+\frac{M_{y,Ed}+\Delta M_{y,Ed}}{M_{y,V,Rd}}+\frac{M_{z,Ed}+\Delta M_{z,Ed}}{M_{z,V,Rd}}\leq 1.0$$

With:

- N_{Rd} determined by pure tension or compression
- $M_{y,Rd}$ en $M_{z,Rd}$ determined with $\rho=\left(2\cdot\sqrt{\frac{V_{y,Ed}}{V_{y,pl,Rd}}^2+\frac{V_{z,Ed}}{V_{z,pl,Rd}}^2}-1\right)^2$
- $\Delta M_{y,Ed}=\Delta M_{z,Ed}=0$ if cross section class is 3, otherwise $\Delta M_{y,Ed}=e_{N,y}N_{Ed}$ and $\Delta M_{z,Ed}=e_{N,z}N_{Ed}$

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2.4. Stability check

- EN 1993-1-1 §6.3.1 Bars loaded with compression (buckling)

$$\frac{N_{Ed}}{N_{b,i,Rd}}\leq 1,0$$

Class 1,2 and 3	Class 4
$N_{b,i,Rd}=\chi_i\cdot A\cdot f_y$	$N_{b,i,Rd}=\chi_i\cdot A_{eff}\cdot f_y$

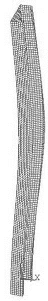
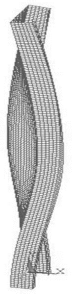
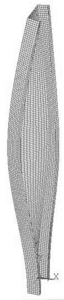
$$\chi_i=\frac{1}{\Phi_i+\sqrt{\Phi_i^2-\bar{\lambda}_i^2}}\leq 1.0$$
$$\Phi_i=0,5\left[1+\alpha_i(\bar{\lambda}_i-0,2)+\bar{\lambda}_i^2\right]$$
$$\bar{\lambda}_i=\sqrt{\frac{A_{eff}\cdot f_y}{N_{cr,i}}}$$

α_i, $N_{cr,i}=ft(L_{cr,i})$, $L_{cr,i}$ en $L_{cr,LT}$ are defined as follows:

- α_i imperfection factors for buckling in accordance with BS EN 1993-1-1 Table 6.1 and 6.2
- $L_{cr,i}$ is the relevant buckling length
- $L_{cr,LT}$ is the relevant lateral torsional buckling length
- $N_{cr,i}$ is the elastic critical force for the relevant buckling mode based on the gross cross sectional properties[5]:

Bending buckling	**Torsional buckling**	**Bending torsion buckling**

[5] NBN EN 1993-1-1 Appendix E

 $N_{cr,i} = \frac{\pi^2 E I_i}{L_{cr,i}^2}$	 $N_{cr,T} = \frac{A}{I_y + I_z}\left(GI_t + \frac{\pi^2 E I_w}{L_{cr,LT}^2}\right)$	 $N_{cr,TF} = \frac{1}{\frac{I_y + I_z}{A} + y_0^2 + z_0^2} \cdot \frac{1}{y_0\left(\frac{I_{yr^2}}{I_z} - 2y_0\right) + z_0\left(\frac{I_{yr^2}}{I_z} - 2z_0\right)} \cdot \left(GI_t + \frac{\pi^2 E I_w}{L_{cr,LT}^2}\right)$

- EN 1993-1-1 §6.3.2 Bars loaded with bending (lateral torsional buckling)

$$\frac{M_{Ed}}{M_{b,Rd}} \leq 1.0$$
$$M_{b,Rd} = \chi_{LT} M_{y,Rd}$$

Method 1 to determine χ_{LT}[6]	**Method 2 to determine χ_{LT}**[7]
$\chi_{LT} = \frac{1}{\Phi_{LT} + \sqrt{\Phi_{LT}^2 - \bar{\lambda}_{LT}^2}} \leq 1.0$	$\chi_{LT} = \frac{1}{\Phi_{LT} + \sqrt{\Phi_{LT}^2 - \beta\bar{\lambda}_{LT}^2}} \leq \begin{cases} 1.0 \\ \frac{1}{\bar{\lambda}_{LT}^2} \end{cases}$
$\Phi_{LT} = 0{,}5\left[1 + \alpha_{LT}(\bar{\lambda}_{LT} - 0{,}2) + \bar{\lambda}_{LT}^2\right]$	$\Phi_{LT} = 0{,}5\left[1 + \alpha_{LT}(\bar{\lambda}_{LT} - \bar{\lambda}_{LT,0}) + \beta\bar{\lambda}_{LT}^2\right]$

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_y f_y}{M_{cr}}} \geq 0{,}2$$

$$M_{cr} = C_1 \frac{\pi^2 E I_z}{(k_z L)^2}\left[\sqrt{\left(\frac{k_z}{k_w}\right)^2 \frac{I_w}{I_z} + \frac{(k_z L)^2 G I_t}{\pi^2 E I_z} + \left(C_2 z_g - C_3 z_j\right)^2} - \left(C_2 z_g - C_3 z_j\right)\right]$$

$\alpha_{LT}, \beta, \bar{\lambda}_{LT,0}, M_{cr} = ft(L, C_1, k_z, k_w), L, C_1, k_z, k_w$ are defined as follows:

- α_{LT} the imperfection factor for lateral torsional buckling according to EN 1993-1-1 Table 6.3, 6.4 and 6.5
- $\beta = 1$, $\bar{\lambda}_{LT,0} = 0{,}2$ for welded sections, $\beta = 0{,}75$ $\bar{\lambda}_{LT,0} = 0{,}4$ for rolled, hot finished and cold-formed hollow sections according to NA to BS EN 1993-1-1 NA.2.17
- M_{cr} is the elastic critical lateral torsional buckling moment according to NBN EN 1993-1-1 Appendix D §2
- C_1 is a factor which takes the moment distribution into account
- The effective length factor k_z relates to the final rotation in plane, k_w relates to the warping of the ends (NBN EN 1993-1-1 Appendix D §2).
- $L_{cr,LT}$ is the relevant lateral torsional buckling length

Class 1 and 2	Class 3	Class 4
$M_{y,Rd} = W_{y,pl} \cdot f_y$	$M_{y,Rd} = W_{y,el} \cdot f_y$	$M_{y,Rd} = W_{y,eff} \cdot f_y$

- EN 1993-1-1 §6.3.3 Bars loaded with compression and bending (buckling + lateral torsional buckling)

$$\frac{N_{Ed}}{\frac{\chi_y N_{Rd}}{\gamma_{M1}}} + k_{yy}\frac{M_{y,Ed} + \Delta M_{y,Ed}}{\frac{\chi_{LT} M_{y,Rd}}{\gamma_{M1}}} + k_{yz}\frac{M_{z,Ed} + \Delta M_{z,Ed}}{\frac{M_{z,Rd}}{\gamma_{M1}}} \leq 1$$
$$\frac{N_{Ed}}{\frac{\chi_z N_{Rd}}{\gamma_{M1}}} + k_{zy}\frac{M_{y,Ed} + \Delta M_{y,Ed}}{\frac{\chi_{LT} M_{y,Rd}}{\gamma_{M1}}} + k_{zz}\frac{M_{z,Ed} + \Delta M_{z,Ed}}{\frac{M_{z,Rd}}{\gamma_{M1}}} \leq 1$$

Method 1 to determine k_{ii}[8]	**Method 2 to determine k_{ii}**[9]

[6] LTB curves according to the general method, EN 1993-1-1 §6.3.2.2
[7] LTB curves according to the equivalent method, EN 1993-1-1 §6.3.2.3
[8] EN 1993-1-1 Appendix A
[9] EN 1993-1-1 Appendix B

Interaction coefficients k_{ii} as a function of the cross-section class, C_{ii} and C_{mi}	Interaction coefficients k_{ii} as a function of the torsional stiffness of the cross section and C_{mi}
Plasticity coefficients C_{ii} as a function of the torsional stiffness of the cross section and C_{mi}	-
Equivalent moment factors C_{mi} as a function of the torsional stiffness of the cross section and $C_{mi,0}$	Equivalent moment factors C_{mi}
Moment factors $C_{mi,0}$	-

Class 1 and 2	Class 3	Class 4
$M_{i,Rd} = W_{i,pl} \cdot f_y$ $\Delta M_{y,Ed} = \Delta M_{z,Ed} = 0$	$M_{i,Rd} = W_{i,el} \cdot f_y$ $\Delta M_{y,Ed} = \Delta M_{z,Ed} = 0$	$M_{i,Rd} = W_{i,eff} \cdot f_y$ $\Delta M_{y,Ed} = e_{N,y} N_{Ed}$ $\Delta M_{z,Ed} = e_{N,z} N_{Ed}$

2.5. Implementation in Diamonds

- The checks in §2.3 and §2.4 are carried out by Diamonds.
- The effect of warping is not taken into account. For I, H and box profiles this approach could be very safe.
- Diamonds presupposes that the cross sections are double symmetrical and that the loads are action in the shear centre of the cross sections, resulting in z_j and $C_2 z_g$ to be zero.

$$M_{cr} = C_1 \frac{\pi^2 E I_z}{(k_z L)^2} \left[\sqrt{\left(\frac{k_z}{k_w}\right)^2 \frac{I_w}{I_z} + \frac{(k_z L)^2 G I_t}{\pi^2 E I_z}} \right]$$

- The factor C_1 is determined according to Equivalent uniform moment factors for lateral- torsional buckling handling or steel members - Journal of constructional Steel Research No 62, 2006; Serna, Lopez, Puente and Young.
- The results of the strength and stability check are displayed as a percentage of the total capacity (= 100%) of the section.
    - A structure is sufficient if both for the strength and stability checks give percentages less than 100%.
    - A structure is NOT sufficient if the strength and/or stability check give percentages more than 100%.

  Based on the percentage no judgment can be made on the extent to which a section is sufficient or not. This is because the formulas used §2.3 and §2.4 are not always linear.
- The intermediate results can be retrieved by double-clicking a bar while the results for the strength or stability verification are displayed.

3. Verification in SLS

Diamonds calculates the displacement, deformations and vibrations, but it is up to user to evaluate them since the limits are project dependent.

4. References

- *Krachtswerking, Grondslagen voor het berekenen en toetsen van staalconstructies voor gebouwen volgens Eurocode 0, 1 en 3, H.H. Snijder en H.M.G.M. Steenbergen, Bouwen met staal, 2011, ISBN 978-90-72830-87-6*
- *Formulaire de résistance des matériaux, Y. Xiong, Eyrolles, ISBN 978-2-212-00525-7*
- *EN 1993-1-1:2005+AC 2009*